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Abstract Title: Fast Alternating Fitting Methods for Trigonometric Curves for Large Data

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Abstract

Problems that require fitting circles, ellipses, and curves of other shapes such as dumbbell curves to data points in the plane arise in many application areas such as pattern recognition, computer vision, statistics, and data analysis; see, e.g., [1, 2, 3, 4, 5]. Available methods for fitting circles or ellipses are very sensitive to outliers in the data, and are time consuming when the number of data points is large. To the best of our knowledge, dumbbell curve fitting methods have not been considered in the literature. To fit such a curve, we introduce an alternating method where in each iterate, a regularized minimization problem is solved. This talk focuses on curve fitting methods that are attractive to use when the number of data points is large. It is concerned with minimization problems that arise when fitting curves in the (x, y)-plane defined in the parametric form

$$\begin{cases} x(t) = \sum_{j=1}^{3}(a_{2j-1,1}\cos(jt) + a_{2j,1}\sin(jt)) + x_0, \\ y(t) = \sum_{j=1}^{3}(a_{2j-1,2}\cos(jt) + a_{2j,2}\sin(jt)) + y_0, \end{cases}$$

where $-\pi < t \leq \pi$, (x_0, y_0) and the $a_{ij} \in \mathbb{R}$ are respectively the center and coefficients to be determined with $a_{11}, a_{22} > 0$. The methods introduced in this talk are not sensitive to errors in the data points. Methods for fitting circles and ellipses efficiently minimize the sum of the squared geometric distances between the given data points and the fitted curves. The techniques developed here can be applied to fitting other kinds of curves as well.

References

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